Map Projections for the Layman - *Stefan A. Voser*

From the Earth surface to its representation on a Map

The morphological shape of the Earth as well as the coverage of the Earth surface are geometrically very complex and not easy to describe. A modern form for describing the Earth surface and the geographic situation on it are topographic maps and more generically geographic or thematic maps as well as their functional extention: digital geodata. One of the basic concepts for describing geographic positions are coordinates and their underlying coordinate reference systems (CRS). Various coordinate reference systems exist in which a geographic location may be described mathematically by coordinates. These systems vary in type, underlying concept and method as well as their instantiation. In each system, the position gets its own coordinate values. These values may differ in a conceptual, mathematical and numerical sense, but represent the same geographic position.

When collecting data stored in different coordinate reference systems, each CRS definition must be known together with its geometric relationship to a standard system. Only then it is possible to transfer all data into a standard coordinate reference system.

There exist several mathematical concepts to describe geographic positions by coordinates. For the following discussion, only two concepts of coordinate reference systems will be mentioned (Figure 2):

- geographic coordinates on mathematical (geodetic) Earth models
- "flattened" coordinates in the plane of a map projection.

Geodetic reference systems are used for describing the figure of the Earth and positions on it: *ellipsoids* (and the *sphere*) are used for describing the horizontal position, whereas *geoids* and other gravity related models are the main reference systems for the elevation. Geodetic reference systems have a *datum*, describing the position and orientation of the model in relation to the Earth and its surface. For the concepts of geodetic reference systems see e.g. [Annoni/Luzet 2000 p14f, p50f, Voser 2000].

Map projections are used to map the curved surface of an ellipsoid onto a plane. They have various characteristics, e.g. mathematical properties as different metric deformations, or they are validated for specific geographical extent etc.

For a long time, coordinate reference systems were only considered by specialists in geodesy and cartography, but since Geographic Information Systems (GIS), the Global Positioning System (GPS) as well as Remote Sensing have a rapidly growing amount of users, these concepts become more and more important: if these concepts and instantiations are not correctly considered, this may result in positional errors of hundreds of meters or up to kilometres or more.

Flattening the Earth

Map projections are used to map the surface of a mathematical Earth model like a sphere or ellipsoid onto a plane based on geometrical or mathematical rules, principles or constraints.

Map projections have *advantages* for calculating geometric properties of spatial entities compared to the calculations of these properties on a curved Earth model. In the plane of the map projection, the calculation of distances, angles, directions and areas may be made based on the rules of classical geometry (Euclidean geometry). On the other hand, the *disadvantages* of map projections are their geometric distortions which depend on the position together with the projection method, its instantiation and implementation. This results in the fact that it is not possible to map from a curved surface like a sphere or spheroid onto a plane without distortions.

The analysis of the deformations is done by applying principles of differential geometry: the laws of surface theory. There, its first fundamental treats the geometric intrinsics (metrics on surfaces). Thereby, the rules to describe lengths, angles, areas are derived on the Gaussian fundamentals.

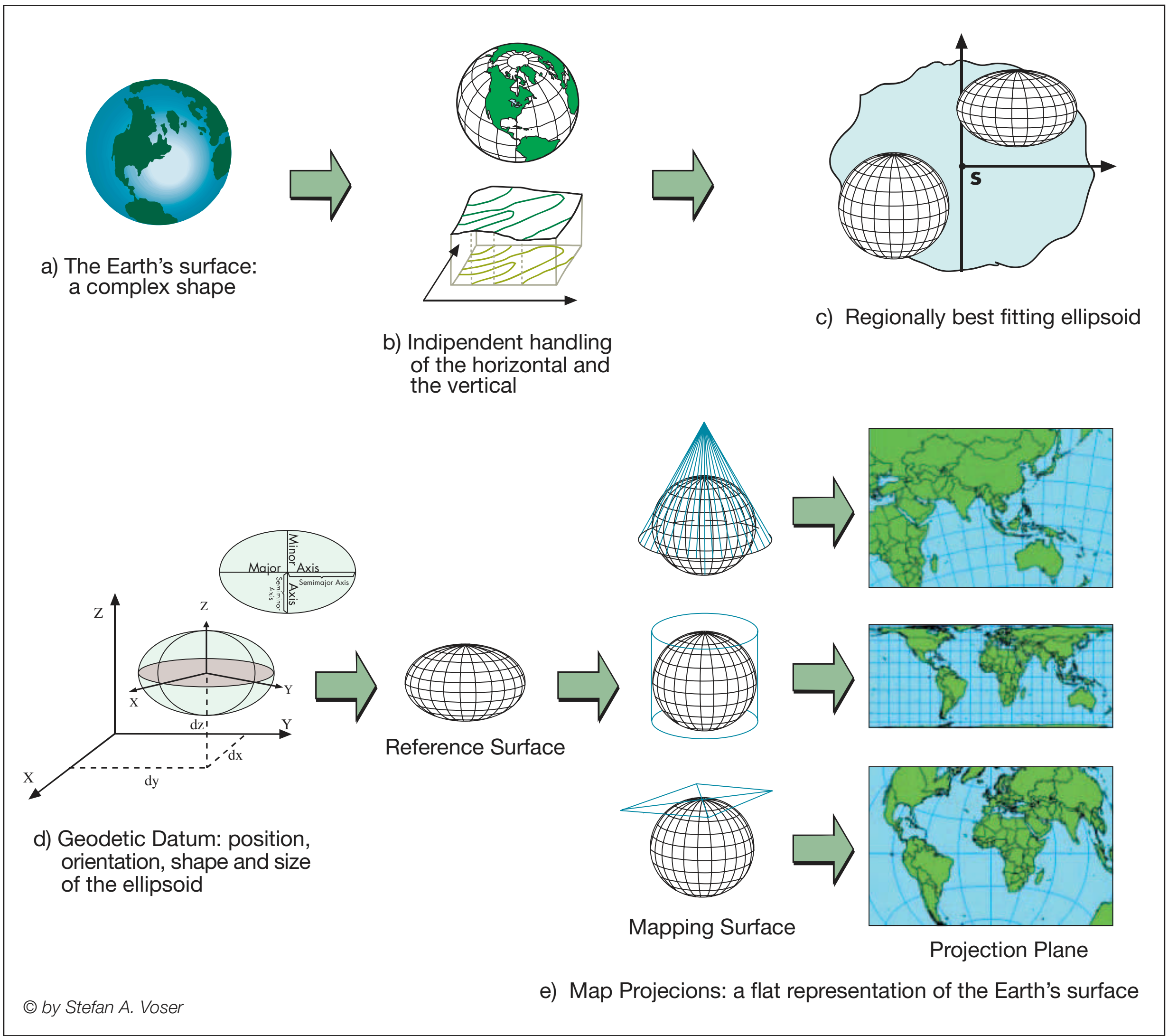


Figure 2:
From the earth surface onto the plane of a map projection.

The analysis of these geometric properties says, it is not possible to map from the surface of a sphere or ellipsoid onto a plane without distortion. Generically, angles, areas and length are distorted. But there exist ways to controll the mentioned deformations in an infinitesian matter.

Because of these distortions, map projections cover a wide field in mathematical cartography, or moreover, in geomatics. Several different types of map projections are known, and already the Ancient Greeks dealt this topic.

There exist various ways to classify map projections:

- the nature of the *mapping surface* (extrinsics of geometry)
- the *distortion* properties (intrinsics of geometry)
- the *geographic use* and extent
- other systematics (visual, mathematical properties...; not discussed below)

In the application, there exist much more individual instances of coordinate reference systems of type map projection. They vary not only in distortion properties, but also in their parameters as well as their method implementations. Important to know when working with map projections is the underlying Earth model and its geodetic datum.

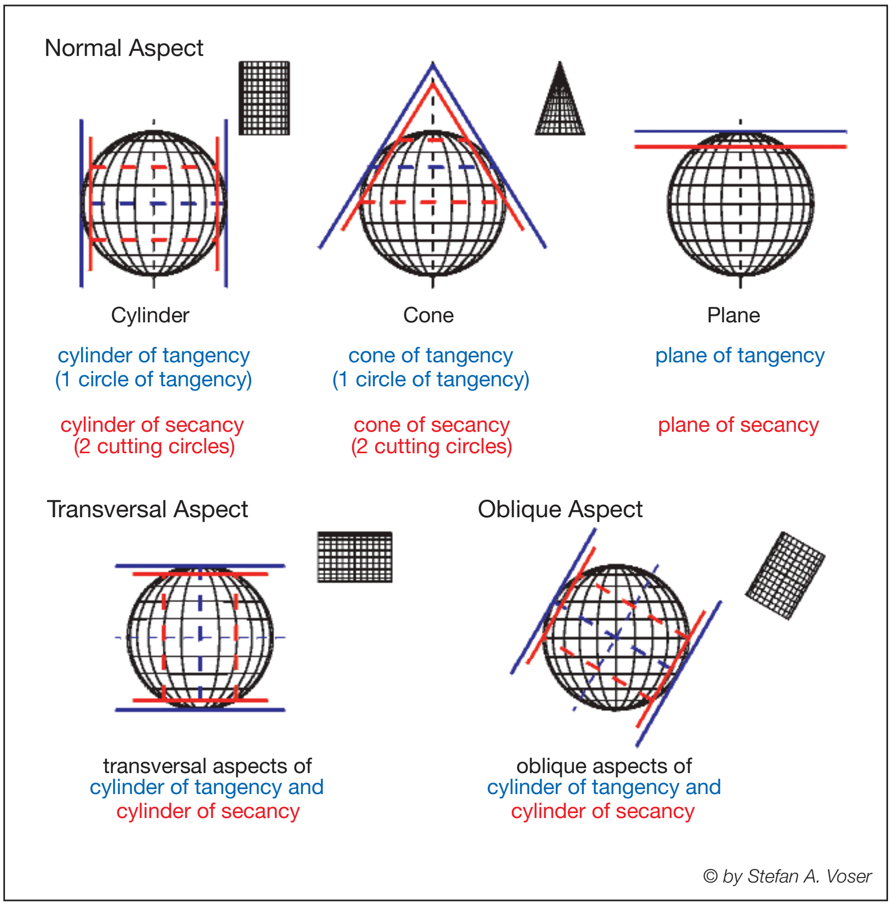


Figure 3: *The mapping surface, their aspects and coincidence.*

Mapping Surfaces

As already declared, a planar representation of the Earth has deformations compared to its shape on the Earth surface. These deformations depend on its position and the nature and specification of the map projection instance. One way to minimise the deformation properties is to use an appropriate mapping surface like a cylinder, cone or horizontal plane and its aspect (alignment) as well as its coincidence with the Earth model. These characteristics are called the geometric extrinsics of the mapping surface (see also Figure 3): nature, aspect and coincidence.

The nature of the *mapping surface*:

- cylinder
- cone
- plane
- polysuperficiality (a continuous system of mapping surfaces).

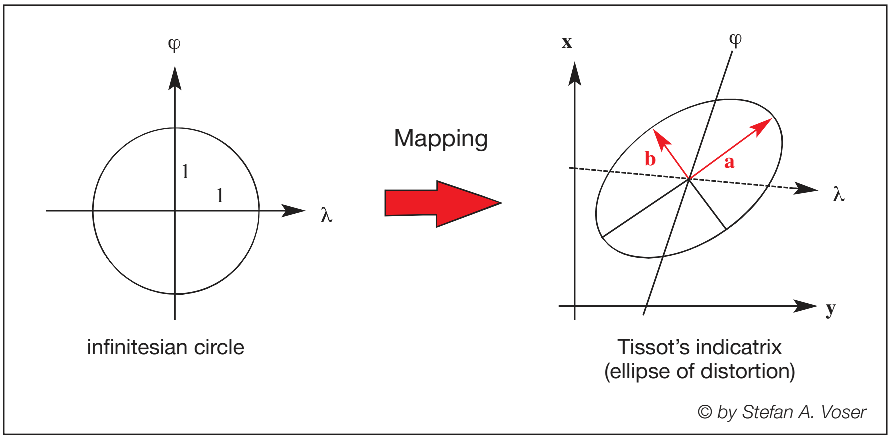
These mapping surfaces may be *aligned* in different ways. The name of its aspect is given based on the orientation of the axis of the mapping surface with the axis if the Earth. In literature, also other terms are used (see e.g. [Goussinsky 1951, Lee 1944, Richardus/Alder 1974, Snyder 1987]):

- *normal* or *direct* aspect (axis parallel to the Earth axis)
- *transversal* aspect (axis parallel to the equator plane)
- *oblique* aspect (axis with any direction).

The third extrinsic category is the *coincidence* (the "contact") of the mapping surface with the underlying Earth model:

- *tangency* ("touching")
- *secancy* ("intersecting").

Figure 4:
Tissot's Indicatrix.



Distortion Properties of Map Projections

As it is not possible to map from the earth surface to a plane without distortions (intrinsics of geometry), a lot of effort has already been done to analyse the distortion properties. The distortions depend on the mapping surface, its aspect and other mathematical or geometrical properties of map projections and are a function of the position. Even though, a specific property may be found which is equal for each position on the projection. In fact, many projections were constructed by restrictions on the distortions. The methods therefore are given by the surface theory. The following metric distortions may be given, but the first three properties exclude each other:

- *conformity* or *orthomorphism* (locally no angular distortion),
- *equivalency* or *authalicity* (locally equal-area properties),
- partially *equidistant* (specific lines as meridians are mapped with true length),
- *compromise* or *error minimised* (restrictions to all distortion properties).

The mathematical instrument to calculate distortions is based on the Tissot Indicatrix: the first order approximation of the mapped shape of an infinitesian small circle on the origin surface is a ellipse, the Tissot Indicatrix (Figure 4). The analysis of this ellipse defines the distortion properties, using the semi major axis a and the semi minor axis b of the ellipse:

- *conformity*: for all points, T.I. is a circle (a=b)
- *equivalency*: for all points, the T.I has the same area (a*b=const)
- partially *equidistant* (specific lines are mapped with same length: l=const)

The use of Map Projections

The amount of different types and variations of map projections existing nowadays has grown to more than 200. (See therefor e.g. Bugayevskiy 1995, Richardus/Adler 1974, Snyder 1987). They vary in properties and usage:

- for the display of *topographic data* mainly conformal projections are used,
- for *thematic and statistical* data very often *equivalent* projections are used, or also *equidistant*,
- for *navigation*, *conformal* projections are used. A very often used projection therefor is/was the Mercator projection which also keeps the lines of the same azimuth (loxodrom) as straight lines,
- for very small scale maps in publications and for wall maps, often also composed projections are used.

An overview of popular map projections is given in the table 1. The main classification is made based on their deformation properties, and their mapping surface. Their main use is given.

Distortion property	Mapping surface	Aspect	Projections	Area of Use (Extent)
conformal	cylinder	normal	Mercator	World equator regions: east-west extent
		transverse	UTM (Universal Transverse Mercator)[18]	a system for the world except polar region (see UPS)
			Transverse Mercator	smaller regions, with north-south extent
			Gauß-Krüger Gauß-Boaga	
		oblique	Rosenmund Oblique Mercator Hotine Oblique Mercator Laborde Oblique Mercator	smaller regions, oblique and east-west extent
	cone	normal	Lambert Conformal Conic	smaller regions, oblique and east-west extent (1 or 2 standard parallels)
	plane	any aspect	stereographic	small regions up to hemisphere
		polar	UPS (Universal Polar Stereographic)	polar regions, (other regions see UTM)
equal area	cylinder	normal	cylindrical equal area by Lambert	equatorial with east-west extent
	pseudo cylinder		Eckert IV Eckert VI Mollweide Sinusoidal	World
	cone	normal	Albers equal area	smaller regions and continents with east-west extent
	pseudo cone		Bonne	smaller regions, east-west extent
	plane	any aspect	Lambert Azimuthal Equal Area	smaller regions, about same north-south, east-west extent
		mainly equatorial	Hammer-Aitoff	World
equidistant	cylinder	normal	Platt (Plate Carrée)	World
		transverse	Cassini Soldner	locally used for large scale mapping
	cone	normal	equidistant conic	smaller regions and continents with (1 or two standard parallels) east-west extent
	plane	any aspect	azimuthal equidistant	smaller regions, about same north-south, east-west extent
others	poly-superficial	normal	Polyconic	locally used for large scale mapping

Table 1:
Overview about often used map projections together with their extential main use.

Projections and Change of Datum

Map projections always are related to an underlying Earth model and its datum. This has to be considered (see Figure 5). If not considering the correct datum, it may affect enormous errors in the horizontal position (hundreds of meters up to more than 1 km). A lot of projections e.g. for world mapping are only used or implemented in current GIS tools on the spherical projections. Thereby, the proper transformation between the ellipsoid and the sphere has to be considered.

[18] has ambiguous implementations, varying in zone definition and underlying datum and ellipsoid.

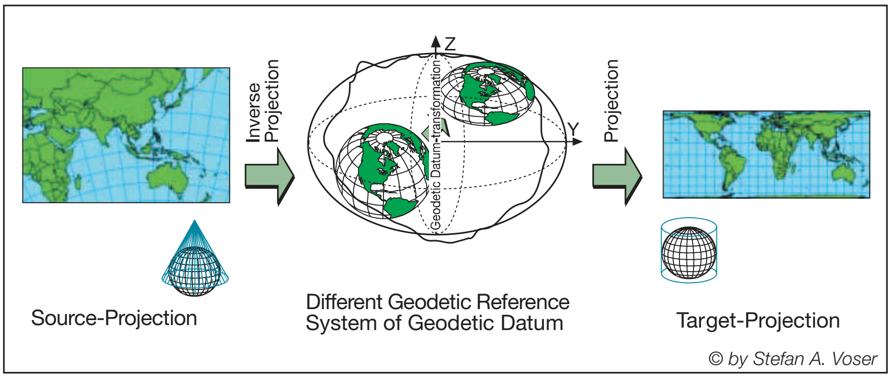


Figure 5:
The change between two map projections is related to the underlying Earth model and its datum. Therefore the geodetic datum transformation has to be considered.

Conclusions

Map projections are used for a flat representation of the spheroidal figure of the Earth. When deciding to instantiate a projection for a purpose, various geometric analyses have to be made, and normally, they should be compared with alternative projections. The instantiation of a projection on one hand depends on the geometric properties and the portrayal of the graticul. On the other hand, also its technical implementation for applying them for digital geospatial data and their processing has to be considered as well. Map projections are an important subject of a comprehensive coordinate reference system management (CRSM) (See e.g. [Voser 1998, Voser 2002]).

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