swisstopo

Bundesamt für Landestopografie
Office fédéral de topographie
Ufficio federale di topografia
Federal Office of Topography

Seftigenstrasse 264
CH-3084 Wabern
Telefon +41 31 963 21 11
Telefax +41 31 963 24 59

Formulas and constants for the calculation of the Swiss conformal cylindrical projection and for the transformation between coordinate systems

May 2006

1 Basic Information

1.1 Summary of the reference systems and reference frames used in Switzerland

System	Frame	Ellipsoid	Map projection
ETRS89	ETRF93	GRS80	(UTM)
CHTRS95	CHTRF95, CHTRF98	GRS80	(UTM, Zone 32)
CH1903	LV03	Bessel 1841	oblique conformal cylindrical
CH1903+	LV95	Bessel 1841	oblique conformal cylindrical

The 3D-reference system **CHTRS95** (Swiss Terrestrial Reference System 1995) is tied closely to the European Terrestrial Reference System **ETRS89** and is identical to it at the epoch 1993.0. Because, until now there are no reasons to change this, the 2 systems will remain identical for some time. CHTRF95 and CHTRF98, the reference frames realised until now, are based on the geocentric-Cartesian coordinates of the fundamental station in Zimmerwald in ETRF93 at the epoch 1993.0.
The local reference system **CH1903+** with its reference frame **LV95** (Landesvermessung 1995) is derived from CHTRS95. In defining CH1903+, it was important that it remains as close as possible to the old reference system CH1903. The parameters defining the system were transferred from the old fundamental station (old observatory of Bern, which does not exist any more) to the new fundamental station in Zimmerwald.
The reference frames LV03 and LV95 show differences of up to 1.6 metres because of the local distortions of LV03. These local distortions are modelled with a local affine transformation (program FINELTRA).

1.2 Height systems used in Switzerland

The official height system **LN02**, which is still in use, was defined in 1902 by fixing the 'height above sea level' of the Repère Pierre du Niton H(RPN)=373.6 m in Geneva, which was obtained from a connection measurement to the tide gauge in Marseilles. The heights of the levelling bench marks were determined by pure levelling. The heights of the nodal points of the 'nivellement de précision' (1864 - 1891) were kept fixed, and the gravity field was not taken into consideration.
The new height system **LHN95** (Landeshöhennetz 1995) is also based on the height of the RPN. However, in this height system the derived geopotential number of the fundamental station in Zimmerwald was declared as the defining constant. The heights of the bench marks of LHN95 are calculated in a kinematic adjustment of the levelling network which takes gravity measurements into account. The user obtains orthometric heights derived from the calculated geopotential numbers.
For exchanging data with neighbouring countries, an additional height system **CHVN95** was defined. It is, for the time being, identical to the European vertical height system EVRS2000. It is based on the height definition of the tide gauge in Amsterdam (NAP) and on the results of the European levelling network (UELN). In this system, height information is exchanged in the form of geopotential numbers and normal heights.

The relationship between the physical heights of LHN95 and CHVN95 with the ellipsoidal heights of CH1903+ and CHTRS95 is guaranteed through the Swiss geoid model **CHGEO98**.

1.3 Reference ellipsoids used in Switzerland

Ellipsoid	Semi-major axis a [m]	Semi-minor axis b [m	Flattening 1/f	1st num. eccentr. e^2
Bessel 1841	6377397.155	6356078.962822	299.15281285	0.006674372230614
GRS 80	6378137.000	6356752.314140	298.257222101	0.006694380023011
WGS 84	6378137.000	6356752.314245	298.257223563	0.006694379990197

Flattening: $$f = \frac{a - b}{a}$$

first numerical eccentricity squared: $$e^2 = \frac{a^2 - b^2}{a^2}$$

1.4 Transformation parameters CHTRS95/ETRS89 ⇔ CH1903(+)

These parameters have been used since 1997 for the transformation between CHTRS95 and CH1903+. Without any restrictions they can also be used for the systems ETRS89 and CH1903. But in the case of CH1903, one must be aware that because of the local distortions of this network the transformed coordinates can be false by up to 1.6 meters compared to the official coordinates in CH1903.

$$\mathbf{X_{CH1903+} = X_{CHTRS95} - 674.374\ m}$$

$$\mathbf{Y_{CH1903+} = Y_{CHTRS95} - 15.056\ m}$$

$$\mathbf{Z_{CH1903+} = Z_{CHTRS95} - 405.346\ m}$$

1.5 Granit87 parameters

These parameters were used between 1987 and 1997 for the transformation between CH1903 and WGS84. We do not recommend their use anymore.

dX = 660.077 m — $\alpha = r_x$ = 2.484 cc (centesimal seconds)
dY = 13.551 m — $\beta = r_y$ = 1.783 cc (centesimal seconds)
dZ = 369.344 m — $\gamma = r_z$ = 2.939 cc (centesimal seconds)
s = 1.00000566 (m = 5.66 ppm)

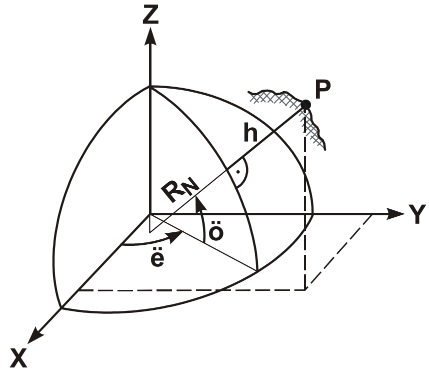
to be used with the transformation formulas:

$$\begin{pmatrix} X_{WGS84} \\ Y_{WGS84} \\ Z_{WGS84} \end{pmatrix} = \begin{pmatrix} dX \\ dY \\ dZ \end{pmatrix} + s \cdot D \cdot \begin{pmatrix} X_{CH1903} \\ Y_{CH1903} \\ Z_{CH1903} \end{pmatrix}$$ with rotation matrix $$D = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$$ and its elements

$$\begin{aligned}
r_{11} &= \cos\beta \cos\gamma \\
r_{21} &= -\cos\beta \sin\gamma \\
r_{31} &= \sin\beta \\
r_{12} &= \cos\alpha \sin\gamma + \sin\alpha \sin\beta \cos\gamma \\
r_{22} &= \cos\alpha \cos\gamma - \sin\alpha \sin\beta \sin\gamma \\
r_{32} &= -\sin\alpha \cos\beta \\
r_{13} &= \sin\alpha \sin\gamma - \cos\alpha \sin\beta \cos\gamma \\
r_{23} &= \sin\alpha \cos\gamma + \cos\alpha \sin\beta \sin\gamma \\
r_{33} &= \cos\alpha \cos\beta
\end{aligned}$$

2 Conversion between ellipsoidal and geocentric-Cartesian coordinates

2.1 Ellipsoidal coordinates (longitude λ, latitude φ, height h) ⇒ geocentric-Cartesian coordinates X, Y, Z



$$X = (R_N + h) \cdot \cos\varphi \cdot \cos\lambda$$
$$Y = (R_N + h) \cdot \cos\varphi \cdot \sin\lambda$$
$$Z = \left(R_N \cdot (1 - e^2) + h\right) \cdot \sin\varphi$$

with normal radius of curvature: $R_N = \frac{a}{\sqrt{1 - e^2 \sin^2\varphi}}$

The parameters a and e are dependent on the reference ellipsoid:

a = semi-major axis of the reference ellipsoid
b = semi-minor axis of the reference ellipsoid

e = first numerical eccentricity of the ellipsoid = $\frac{\sqrt{a^2 - b^2}}{a}$

2.2 Geocentric-Cartesian coordinates X, Y, Z ⇒ ellipsoidal coordinates (longitude λ, latitude φ, height h)

$$\lambda = \arctan\left(\frac{Y}{X}\right) \qquad \varphi = \arctan\left(\frac{\frac{Z}{\sqrt{X^2 + Y^2}}}{1 - \frac{R_N \cdot e^2}{R_N + h}}\right) \qquad h = \frac{\sqrt{X^2 + Y^2}}{\cos\varphi} - R_N$$

with $R_N = \frac{a}{\sqrt{1 - e^2 \sin^2\varphi}}$

Please note: The quantities φ, R_N and h are dependent on each other. Therefore, they have to be calculated by iteration (starting with an approximate value φ_0):

Proposed value for φ_0: $\varphi_0 = \arctan\frac{Z}{\sqrt{X^2 + Y^2}}$

3 Swiss projection formulas

3.1 Notation, constants, auxiliary values

Notation

φ, λ: ellipsoidal latitude and longitude in the system CH1903/03+ relative to Greenwich
b, l : spherical coordinates relative to Bern
$\bar{b}, \bar{l}$: spherical coordinates relative to the pseudo-equatorial system in Bern
Y, X : civilian projection coordinates
y, x : official (military) projection coordinates in LV03 or LV95

Where not indicated, the units for angles are radians [rad] and the units for lengths are meters [m] in all formulas.

Constants

a	=	6377397.155 m	semi-major axis of the Bessel-ellipsoid
E^2	=	0.006674372230614	1st numerical eccentricity (squared) of the Bessel ellipsoid (*)
φ_0	=	46° 57' 08.66"	ellipsoidal latitude of the projection centre in Bern (**)
λ_0	=	7° 26' 22.50"	ellipsoidal longitude of the projection centre in Bern (**)

(*) In order to distinguish it from the Euler constant e, the 1st numerical eccentricity in these formulas is noted as E.

(**) These are the so-called 'old values', which are still valid for all geodetic purposes. The so-called 'new values' (from a new determination of the astronomical coordinates of the fundamental station in Bern from 1938: φ_0 = 46° 57' 07.89", λ_0 = 7° 26' 22.335") have only been used for cartographic purposes (indication of latitudes and longitudes on the national maps). We do not recommend the use of these values.

Calculation of auxiliary values

Radius of the projection sphere:

$$R = \frac{a \cdot \sqrt{1-E^2}}{1-E^2 \sin^2 \varphi_0} \qquad = 6378815.90365 \text{ m}$$

Relat. between longitude on sphere and on ellipsoid:

$$\alpha = \sqrt{1 + \frac{E^2}{1-E^2} \cdot \cos^4 \varphi_0} \qquad = 1.00072913843038$$

Latitude of the fundamental point on the sphere:

$$b_0 = \arcsin\left(\frac{\sin \varphi_0}{\alpha}\right) \qquad = 46° \ 54' \ 27.83324844"$$

Constant of the latitude formula:

$$K = \ln\left(\tan\left(\frac{\pi}{4} + \frac{b_0}{2}\right)\right) - \alpha \cdot \ln\left(\tan\left(\frac{\pi}{4} + \frac{\varphi_0}{2}\right)\right) + \frac{\alpha \cdot E}{2} \cdot \ln\left(\frac{1 + E \cdot \sin \varphi_0}{1 - E \cdot \sin \varphi_0}\right) \qquad = 0.0030667323772751$$

3.2 Ellipsoidal coordinates (λ, φ) ⇒ Swiss projection coordinates (y, x) (rigorous formulas)

The numerical calculation is performed for the station Rigi with the following values:

φ = 47° 03' 28.95659233" = 0.821317799 rad
λ = 8° 29' 11.11127154" = 0.148115967 rad

a) ellipsoid (φ, λ) ⇒ sphere (b, l) (Gauss projection)

Auxiliary value:

$$S = \alpha \cdot \ln\left(\tan\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)\right) - \frac{\alpha \cdot E}{2} \cdot \ln\left(\frac{1+E\cdot\sin\varphi}{1-E\cdot\sin\varphi}\right) + K$$

= 0.931969601072417

spherical latitude:

$$b = 2\cdot(\arctan(e^{S}) - \frac{\pi}{4})$$

= 0.820535226 rad
(= 47° 00' 47.539422864")

spherical longitude:

$$l = \alpha\cdot(\lambda - \lambda_0)$$

= 0.0182840649 rad
(= 1° 02' 51.3591108468")

b) equator system (b, l) ⇒ pseudo-equator system ($\bar{b}$, $\bar{l}$) (rotation)

$$\bar{l} = \arctan\left(\frac{\sin l}{\sin b_0\cdot\tan b + \cos b_0\cdot\cos l}\right)$$

= 0.0124662714 rad
(= 0° 42' 51.3530463924")

$$\bar{b} = \arcsin(\cos b_0\cdot\sin b - \sin b_0\cdot\cos b\cdot\cos l)$$

= 0.00192409259 rad
(= 0° 06' 36.8725855284")

c) sphere ($\bar{b}$, $\bar{l}$) ⇒ projection plane (y, x) (Mercator projection)

$$Y = R\cdot\bar{l}$$

= 79520.05
y_{LV03} = Y + 600000 = 679520.05
y_{LV95} = Y + 2600000 = 2679520.05

$$X = \frac{R}{2}\cdot\ln\left(\frac{1+\sin\bar{b}}{1-\sin\bar{b}}\right)$$

= 12273.44
x_{LV03} = X + 200000 = 212273.44
x_{LV95} = X + 1200000 = 1212273.44

3.3 Swiss projection coordinates (y, x) ⇒ ellipsoidal coordinates (λ, φ) (rigorous formulas)

Again the point Rigi was used as an example: y = 679520.05 x = 212273.44

a) projection plane (y, x) ⇒ sphere ($\bar{b}$, $\bar{l}$)

$Y = y_{LV03} - 600'000$	$Y = y_{LV95} - 2'600'000$	= 79520.05
$X = x_{LV03} - 200'000$	$X = x_{LV95} - 1'200'000$	= 12273.44

$$\bar{l} = \frac{Y}{R} \qquad 0.01246627136 \text{ rad}$$

$$\bar{b} = 2 \cdot \left[\arctan\left(e^{\frac{X}{R}} \right) - \frac{\pi}{4} \right] \qquad 0.00192409259 \text{ rad}$$

b) pseudo-equator system ($\bar{b}$, $\bar{l}$) ⇒ equator system (b, l)

$$b = \arcsin\left(\cos b_0 \cdot \sin \bar{b} + \sin b_0 \cdot \cos \bar{b} \cdot \cos \bar{l}\right) \qquad = 0.820535226 \text{ rad}$$

$$l = \arctan\left(\frac{\sin \bar{l}}{\cos b_0 \cdot \cos \bar{l} - \sin b_0 \cdot \tan \bar{b}} \right) \qquad = 0.0182840649 \text{ rad}$$

c) sphere (b, l) ⇒ ellipsoid (φ, λ)

$$\lambda = \lambda_0 + \frac{l}{\alpha} \qquad = 0.148115967 \text{ rad}$$

= 8° 29' 11.111272"

$$S = \ln \tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right) = \frac{1}{\alpha}\left[\ln \tan\left(\frac{\pi}{4} + \frac{b}{2}\right) - K\right] + E \cdot \ln \tan\left(\frac{\pi}{4} + \frac{\arcsin(E \cdot \sin\varphi)}{2}\right)$$

$$\varphi = 2 \arctan\left(e^S\right) - \frac{\pi}{2}$$

The equations for φ and also for S have to be resolved by **iteration**. As a starting value we propose φ = b.

The iteration steps give the following results:

0. step	S = 0	φ = 0.820535226
1. step	S = 0.933114264192610	φ = 0.821315364725524
2. step	S = 0.933117825679560	φ = 0.821317791017021
3. step	S = 0.933117836751434	φ = 0.821317798559814
4. step	S = 0.933117836785854	φ = 0.821317798583263
5. step	S = 0.933117836785961	φ = 0.821317798583336
6. step	S = 0.933117836785961	φ = 0.821317798583336
		φ = 47° 03' 28.956592"

3.4 Swiss projection coordinates (y, x) ⇒ ellipsoidal coordinates (φ, λ) (approximate formulas)

simplified from: [Bolliger 1967]

Notation and units

φ, λ	=	ellipsoidal latitude and longitude in the system CH1903/03+ relative to Greenwich **in [10000 "]**
Y, X	=	civilian projection coordinates **in [1000 km]**
y, x	=	official (military) projection coordinates in LV03 or LV95 **in [1000 km]**

Calculation

$Y = y_{LV03} - 0.6$ $\quad$ $X = x_{LV03} - 0.2$ resp.

$Y = y_{LV95} - 2.6$ $\quad$ $X = x_{LV95} - 1.2$

$\lambda = 2.67825 + a1*Y + a3*Y^3 + a5*Y^5$ with

a1 =		a3 =		a5 =	
+ 4.729 730 56		- 0.044 270		+ 0.000 96	
+ 0.792 571 4	* X	- 0.025 50	* X		
+ 0.132 812	* X^2	- 0.009 6	* X^2		
+ 0.025 50	* X^3				
+ 0.004 8	* X^4				

$\varphi = 16.902866 + p0 + p2*Y^2 + p4*Y^4$ with

p0 =		p2 =		p4 =	
0		- 0.271 353 79		+ 0.002 442	
+ 3.238 648 77	* X	- 0.045 044 2	* X	+ 0.001 32	* X
- 0.002 548 6	* X^2	- 0.007 553	* X^2		
- 0.013 245	* X^3	- 0.001 46	* X^3		
+ 0.000 048	* X^4				

approximation error (for IYI < 0.2 and IXI < 0.1):

approximation to 3rd degree:	Δλ <	0.16"	and	Δφ <	0.04"
approximation to 5th degree:	Δλ <	0.00014"	and	Δφ <	0.00004"

To control the calculation, the example (point Rigi) of the preceding chapter can be used. Further approximate formulas and examples can be found in [Bolliger 1967].

3.5 Ellipsoidal coordinates (λ, φ) ⇒ Swiss projection coordinates (y, x) (approximate formulas)

simplified from: [Bolliger 1967]

Notation and units

φ, λ = ellipsoidal latitude and longitude in the system CH1903/03+ relative to Greenwich **in [10'000 "]**
Y, X = civilian projection coordinates **in [1000 km]**
y, x = official (military) projection coordinates in LV03 or LV95 **in [1000 km]**

auxiliary values:

$\Phi = \varphi - 16.902866''$

$\Lambda = \lambda - 2.67825''$

Calculation

$Y = y1 \cdot \Lambda + y3 \cdot \Lambda^3 + y5 \cdot \Lambda^5$ with

y1 = + 0.211 428 533 9
- 0.010 939 608 * Φ
- 0.000 002 658 * Φ^2
- 0.000 008 53 * Φ^3

y3 = - 0.000 044 232 7
+ 0.000 004 291 * Φ
- 0.000 000 309 * Φ^2

y5 = + 0.000 000 019 7

$X = x0 + x2 \cdot \Lambda^2 + x4 \cdot \Lambda^4$ with

x0 = 0
+ 0.308 770 746 3 * Φ
+ 0.000 075 028 * Φ^2
+ 0.000 120 435 * Φ^3
+ 0 * Φ^4
+ 0.000 000 07 * Φ^5

x2 = + 0.003 745 408 9
- 0.000 193 792 7 * Φ
+ 0.000 004 340 * Φ^2
- 0.000 000 376 * Φ^3

x4 = - 0.000 000 734 6
+ 0.000 000 144 4 * Φ

$y_{LV03} = Y + 0.6$ $x_{LV03} = X + 0.2$ resp.
$y_{LV95} = Y + 2.6$ $x_{LV95} = X + 1.2$

approximation error (for IΛI < 1.0 and IΦI < 0.316):

approximation to 3rd degree:	ΔY < 1.2 m and	ΔX < 0.75 m
approximation to 5th degree:	ΔY < 0.001 m and	ΔX < 0.0007 m

To control the calculation, the example (point Rigi) of the preceding chapter can be used. Further approximate formulas and examples can be found in [Bolliger 1967].

3.6 Formulas for meridian convergence and for scale distortion

The distortions, caused by the projection, can be described completely by the **meridian convergence μ** (angle between the ellipsoidal north direction and the grid north direction of the projection) and the **scale distortion m** (relationship of an infinitesimally small line in the projection and on the ellipsoid):

meridian convergence:

$$\mu = \arctan \frac{\sin b_0 \cdot \sin l}{\cos b_0 \cdot \cos b + \sin b_0 \cdot \sin b \cdot \cos l}$$

approximate formula:

$$\mu = 10.668 \cdot 10^{-6} \cdot Y + 1.788 \cdot 10^{-12} \cdot Y \cdot X - 0.14 \cdot 10^{-18} \cdot Y^3$$

Y and X denote the projection coordinates in the civilian system in [m]. The meridian convergence μ is obtained in Grads (Gons).

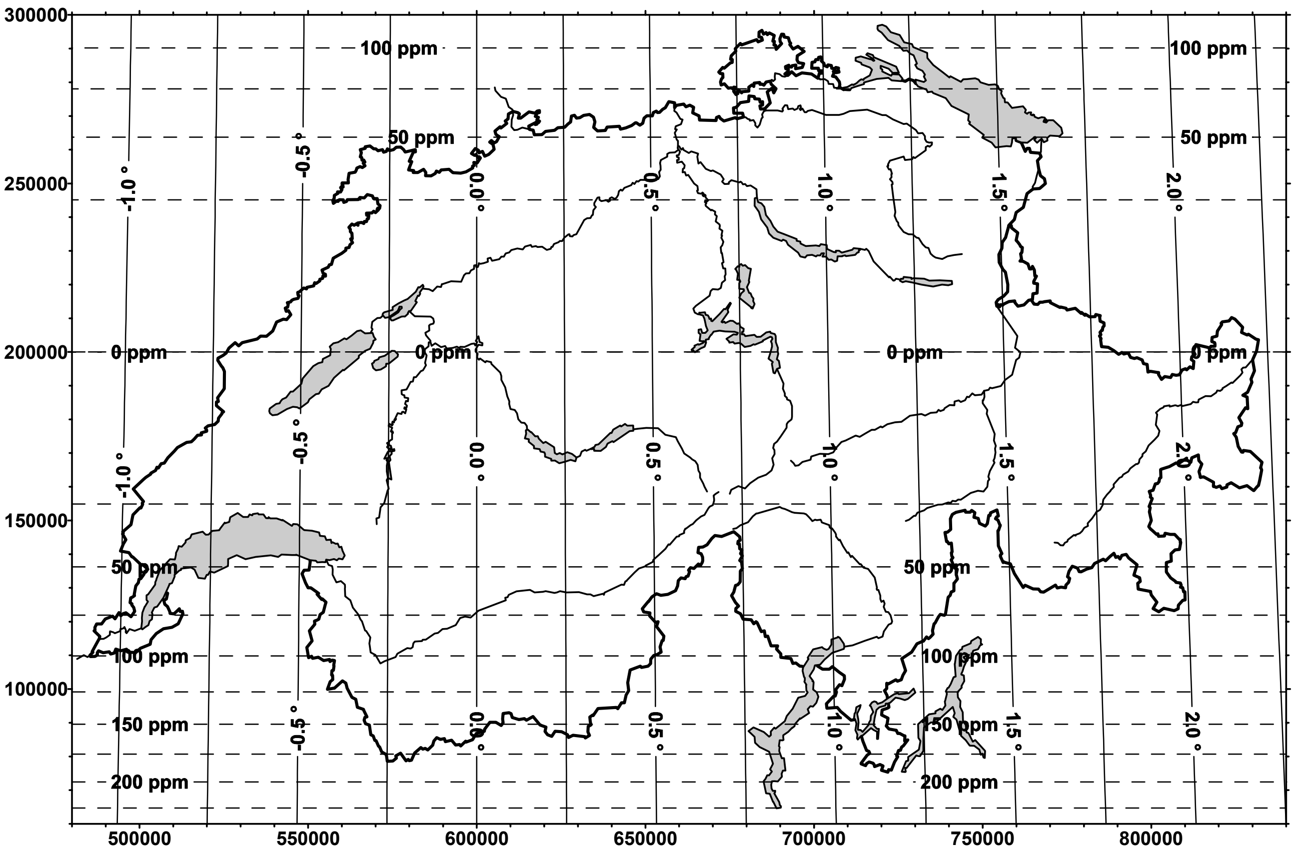
Scale distortion (main term):

$$m = \frac{s_{proj}}{s_{ell}} = \alpha \cdot \frac{R}{R_N} \cdot \frac{\cos b}{\cos \varphi \cdot \cos \bar{b}}$$

approximate formula:

$$m = 1 + \frac{X^2}{2R^2}$$

Example: Point Rigi (y = 679520.05, x = 212273.44)
from geographic coordinates: μ = 0.8499955 gon, m = 1.000001852
from approximative formulas: μ = 0.8499946 gon, m = 1.000001851



Representation of the meridian convergence (in degrees) and of the scale distortion (dashed, in ppm)

4 Approximate solution for the transformation CH1903 ⇔ WGS84

4.1 Approximate formulas for the direct transformation of: ellipsoidal WGS84 coordinates (φ, λ, h) ⇒ Swiss projection coordinates (y, x, h')

(Precision in the order of 1 metre)
These formulas must not be used for cadastral surveying or geodetic applications !

After : [H. Dupraz, Transformation approchée de coordonnées WGS84 en coordonnées nationales suisses, IGEO-TOPO, EPFL, 1992]

The parameters were re-determined by U. Marti (May 1999). In addition, the units were changed so that the parameters are comparable to the values published in [Bolliger 1967].

1. The latitudes φ and longitudes λ have to be converted into arc seconds ["]

2. The following auxiliary values have to be calculated (differences of latitude and longitude relative to the projection centre in Bern in the unit [10000"]):

$$\varphi' = (\varphi - 169028.66\,'')/10000$$
$$\lambda' = (\lambda - 26782.5\,'')/10000$$

3. $$y\,[m] = 600072.37 + 211455.93 \cdot \lambda' - 10938.51 \cdot \lambda' \cdot \varphi' - 0.36 \cdot \lambda' \cdot \varphi'^2 - 44.54 \cdot \lambda'^3$$

$$x\,[m] = 200147.07 + 308807.95 \cdot \varphi' + 3745.25 \cdot \lambda'^2 + 76.63 \cdot \varphi'^2 - 194.56 \cdot \lambda'^2 \cdot \varphi' + 119.79 \cdot \varphi'^3$$

$$h'\,[m] = h - 49.55 + 2.73 \cdot \lambda' + 6.94 \cdot \varphi'$$

4. Numerical example:

given:	φ = 46° 2' 38.87"	λ = 8° 43' 49.79"	h = 650.60 m
⇒	φ' = -0.326979	λ' = 0.464729	
⇒	y = 699 999.76 m	x = 99 999.97 m	h' = 600.05 m
result NAVREF:	y = 700 000.0 m	x = 100 000.0 m	h' = 600 m

The precision of the approximate formulas are better than 1 metre in position and 0.5 metres in height everywhere in Switzerland.

Remark on the heights: In these formulas one is supposed to work with ellipsoidal heights as obtained by GPS measurements. If 'heights above sea level' are used, the heights are the same in both systems on the 1 metre level. Therefore, no transformation is necessary.

4.2 Approximate formulas for the direct transformation of: Swiss projection coordinates (y, x, h') ⇒ ellipsoidal WGS84 coordinates (φ, λ, h)

(Precision in the order of 0.1")
These formulas must not be used for cadastral surveying or geodetic applications !

These formulas were derived by U. Marti in May 1999, based on the formulas in [Bolliger, 1967]

1. The projection coordinates y (easting) and x (northing) have to be converted into the civilian system (Bern = 0 / 0) and have to expressed in the unit [1000 km] :

$$y' = (y - 600000 \text{ m})/1000000$$
$$x' = (x - 200000 \text{ m})/1000000$$

2. The longitude and latitude have to be calculated in the unit [10000"]

$$\begin{aligned} \lambda' = \; & 2.6779094 \\ & + 4.728982 * y' \\ & + 0.791484 * y' * x' \\ & + 0.1306 * y' * x'^2 \\ & - 0.0436 * y'^3 \end{aligned}$$

$$\begin{aligned} \varphi' = \; & 16.9023892 \\ & + 3.238272 * x' \\ & - 0.270978 * y'^2 \\ & - 0.002528 * x'^2 \\ & - 0.0447 * y'^2 * x' \\ & - 0.0140 * x'^3 \end{aligned}$$

$$\begin{aligned} h\,[m] = \; & h' + 49.55 \\ & - 12.60 * y' \\ & - 22.64 * x' \end{aligned}$$

3. Longitude and latitude have to be converted to the unit [°]

$$\lambda = \lambda' * 100 / 36$$
$$\varphi = \varphi' * 100 / 36$$

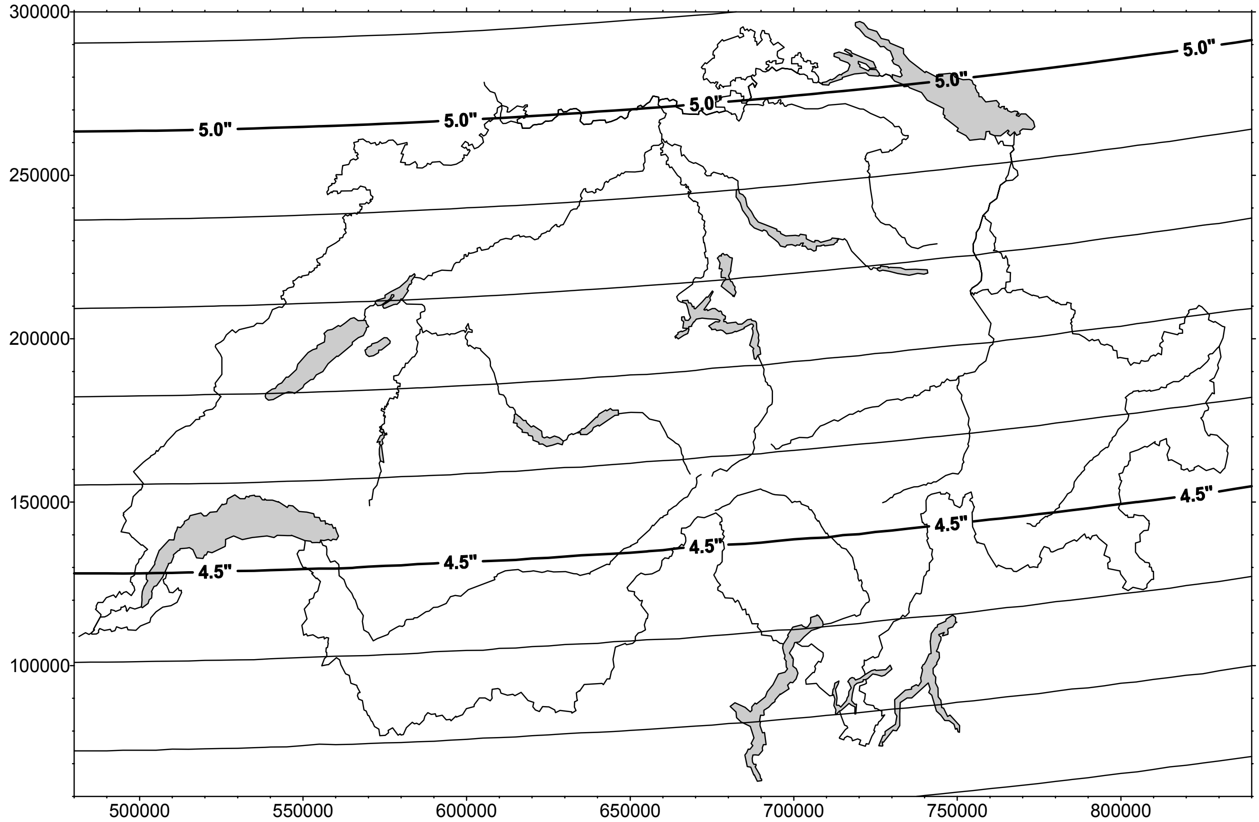
4. Numerical example

given :	y = 700 000 m	x = 100 000 m	h' = 600 m
⇒	y' = 0.1	x' = -0.1	
⇒	λ' = 3.14297976	φ' = 16.57588564	h = 650.55 m
⇒	λ = 8° 43' 49.80"	φ = 46° 02' 38.86"	
result NAVREF:	λ = 8° 43' 49.79"	φ = 46° 02' 38.87"	h = 650.60 m

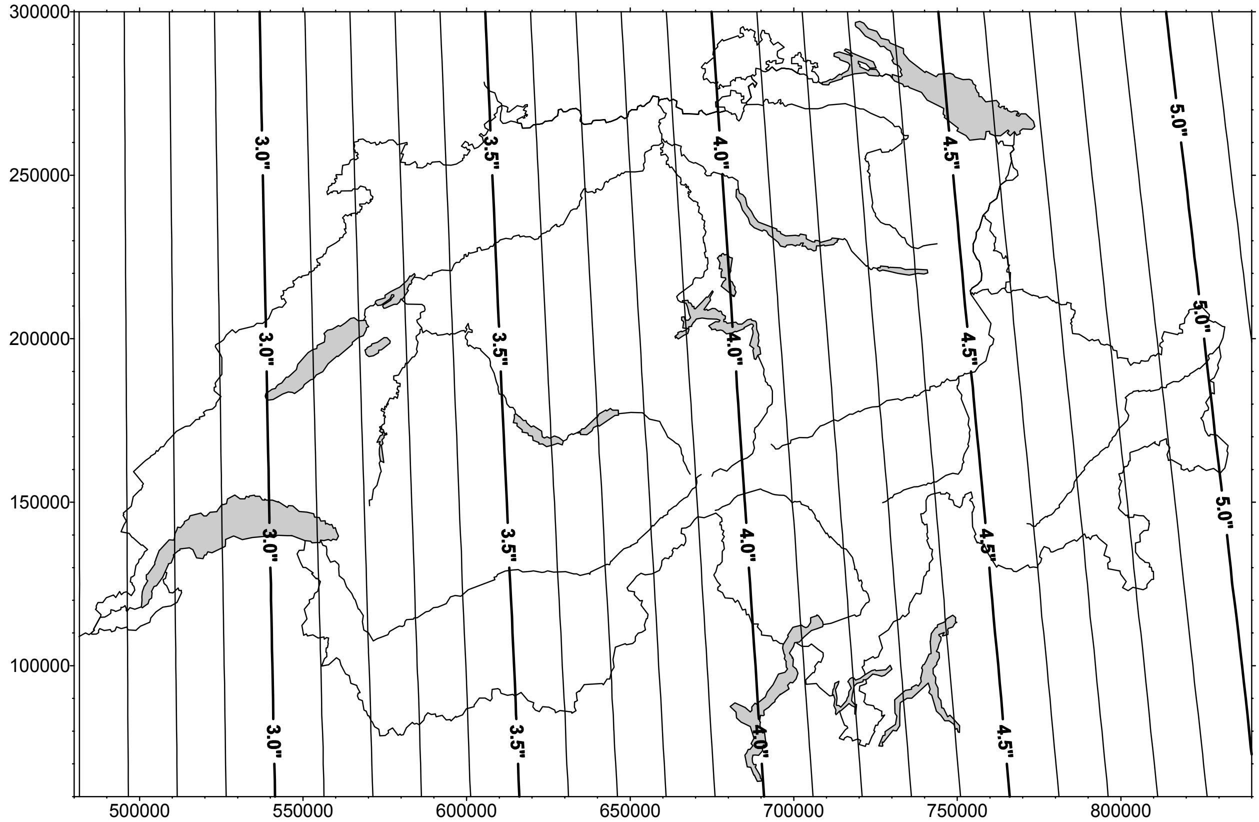
The precision of the approximate formulas is better than 0.12" in longitude, 0.08" in latitude and 0.5 metres in height everywhere in Switzerland.

Remark on the heights: In these formulas one is supposed to work with ellipsoidal heights as obtained by GPS measurements. If 'heights above sea level' are used, the heights are the same in both systems on the 1 metre level. Therefore, no transformation is necessary.

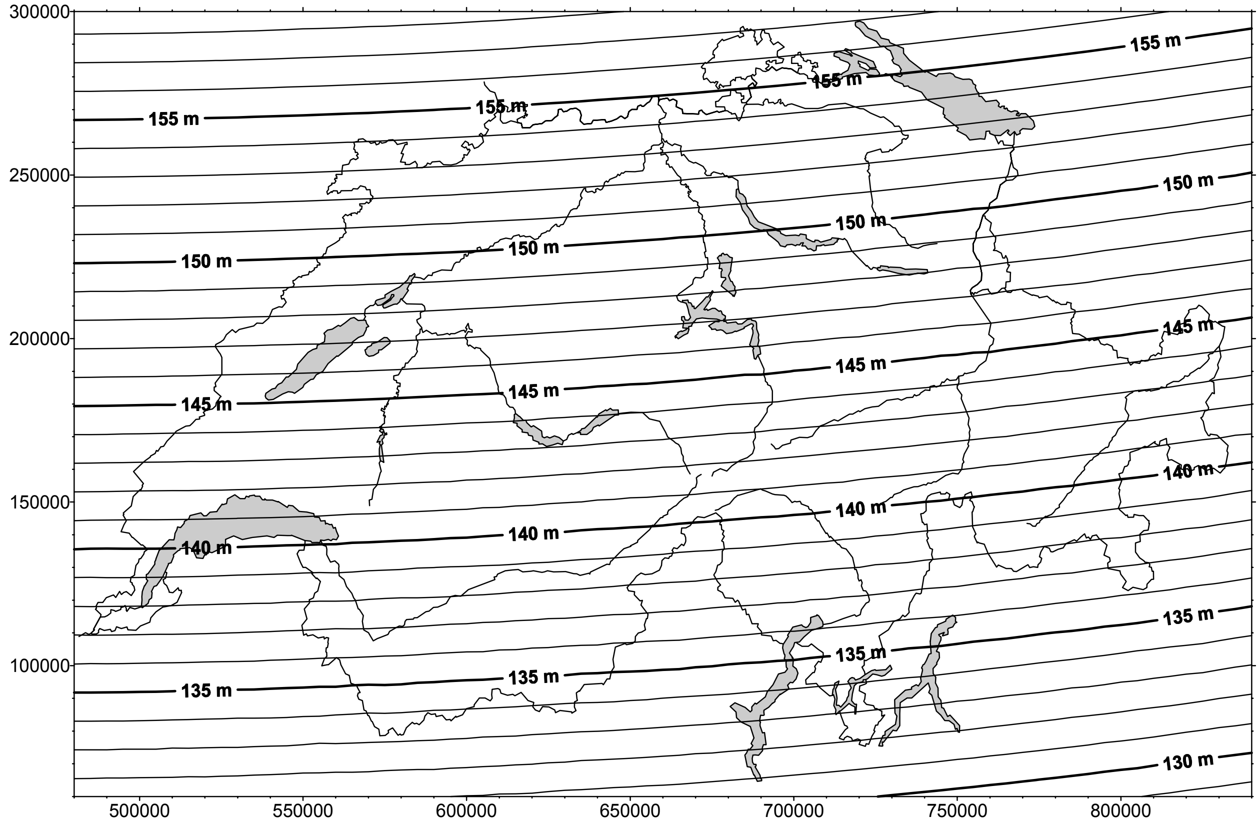
5 Diagrams of the differences between CH1903 and WGS84



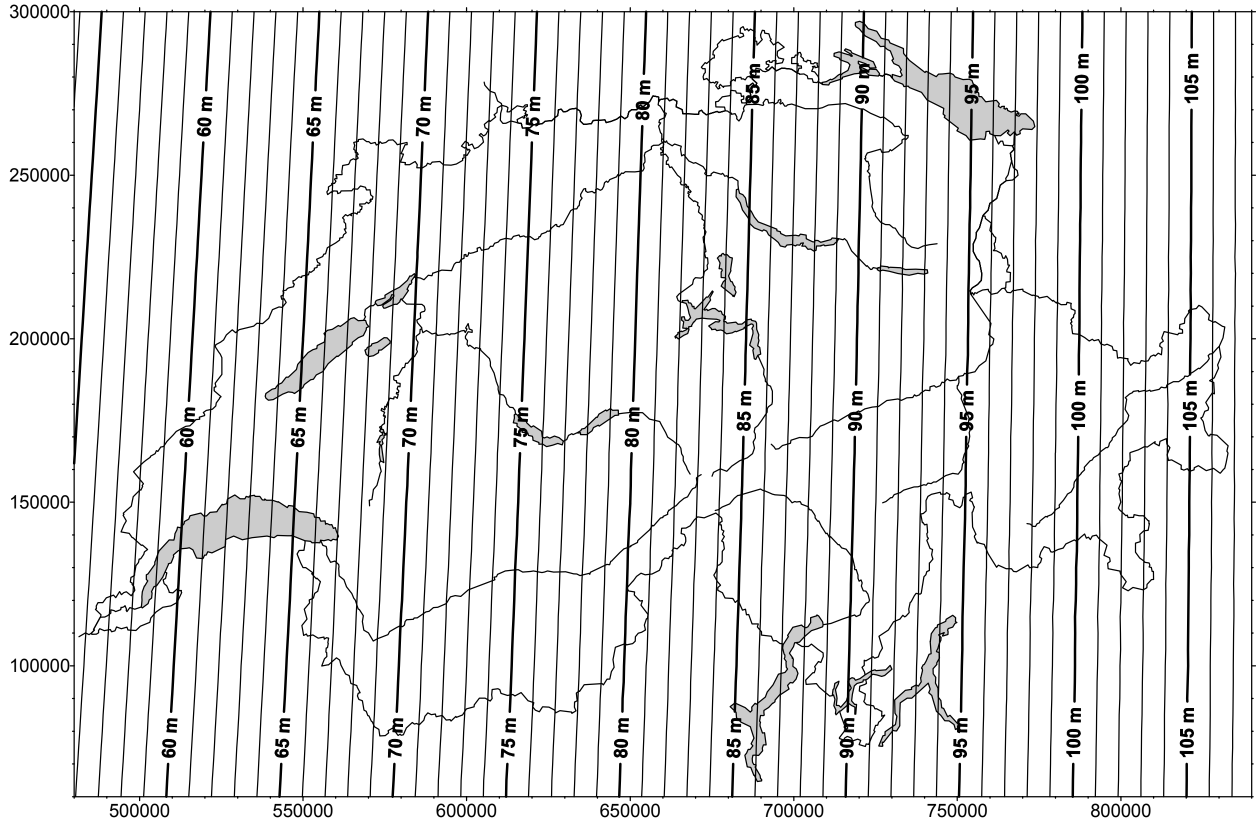
Differences in latitude: CH1903 minus WGS84 (in arc seconds)



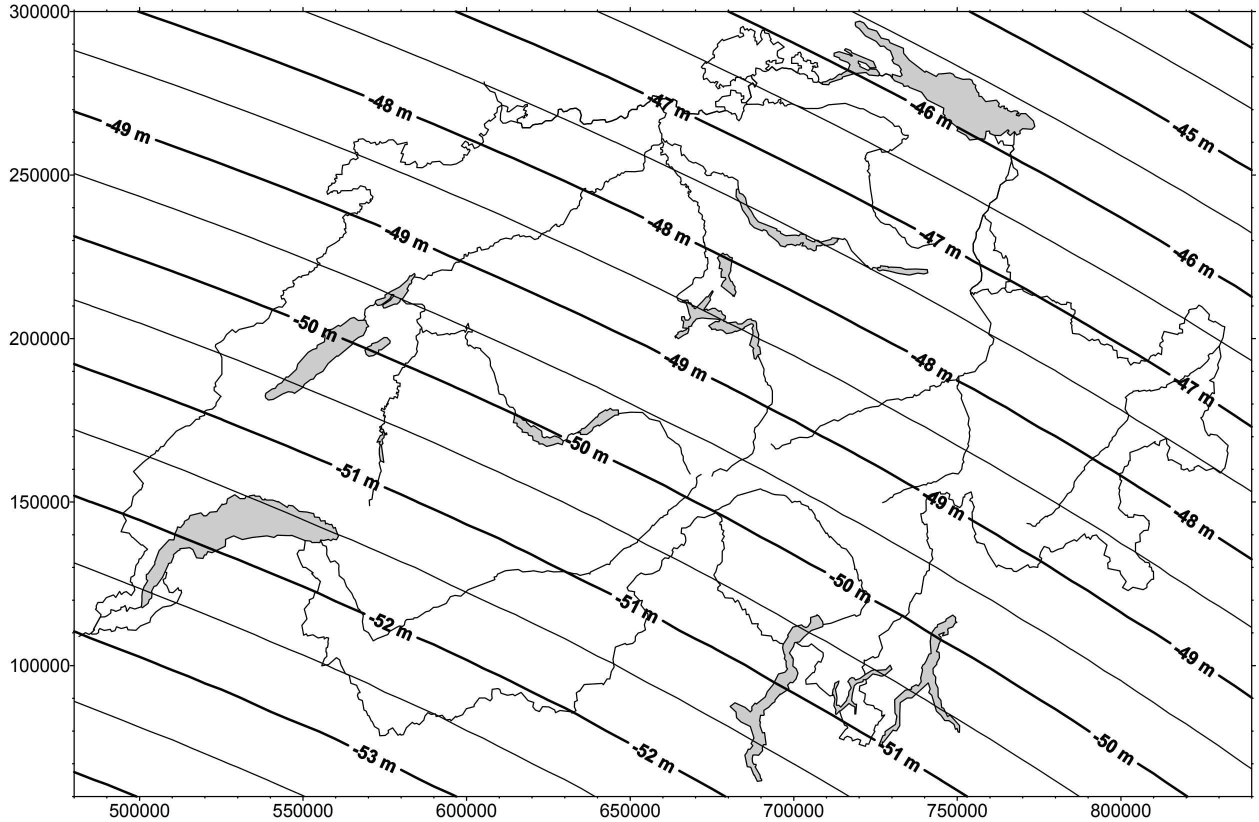
Differences in longitude: CH1903 minus WGS84 (in arc seconds)



Differences in latitude: CH1903 minus WGS84 (converted to metres)



Differences in longitude: CH1903 minus WGS84 (converted to metres)



Differences in height (ellipsoidal): CH1903 minus WGS84 [in metres]

6 Numerical examples

Coordinate transformation LV95 ⇒ ETRF93 (~WGS84)

Swiss projection coordinates LV95 with orthometric heights

```
$$PK LV95.pk                                                  30.03.1999 14:02
Zimmerwald                      2602030.77..1191775.06..     897.9063
Chrischona                      2617306.92..1268507.87..     455.9016
Pfaender                        2776668.59..1265372.25..    1042.5898
La Givrine                      2497312.65..1145626.14..    1207.4658
Monte Generoso                  2722649.39..1087786.37..    1692.9974
```

⇒ Calculation and addition of the geoid undulation (program CHGEO98R) ⇒

Swiss projection coordinates LV95 with ellipsoidal heights

```
$$PE used Geoid Model: CHGEO98R (Marti 1998) LV95             07.04.1999 10:38
Zimmerwald                      2602030.77  1191775.06       897.3627         -.5436 98 CH
Chrischona                      2617306.92  1268507.87       457.1300         1.2284 98 CH
Pfaender                        2776668.59  1265372.25      1043.6200         1.0302 98 CH
La Givrine                      2497312.65  1145626.14      1206.3400        -1.1258 98 CH
Monte Generoso                  2722649.39  1087786.37      1690.6600        -2.3374 98 CH
```

⇒ Conversion to

ellipsoidal coordinates and heights in CH1903+

```
$$EL ellipsoidal coordinates in CH1903+                        07.04.1999 10:44
Zimmerwald                          7 27 58.417745   46 52 42.270255      897.3627
Chrischona                          7 40 10.574820   47 34  6.404965      457.1300
Pfaender                            9 47  8.465984   47 31   .092648     1043.6200
La Givrine                          6  6  9.983811   46 27 19.272743     1206.3400
Monte Generoso                      9  1 15.646553   45 55 54.253090     1690.6600
```

⇒ Conversion to

Geocentric-Cartesian coordinates in CH1903+

```
$$3D geocentric-Cartesian coordinates in CH1903+               07.04.1999 10:45
Zimmerwald                        4330616.71244   567539.79285  4632721.68605
Chrischona                        4272473.55710   575353.23757  4684498.28763
Pfaender                          4252889.17730   733507.30318  4681046.76046
La Givrine                        4377121.12355   467993.58998  4600671.91414
Monte Generoso                    4389438.95988   696869.11597  4560727.60896
```

⇒ Addition of the three transformation parameters: dX = + 674.374 m, dY = + 15.056 m, dZ = + 405.346 m ⇒

Geocentric-Cartesian coordinates in ETRF93 / CHTRF95

```
$$3D geocentric-Cartesian coordinates in ETRF93           07.04.1999 10:42
Zimmerwald                        4331291.08644   567554.84885  4633127.03205
Chrischona                        4273147.93110   575368.29357  4684903.63363
Pfaender                          4253563.55130   733522.35918  4681452.10646
La Givrine                        4377795.49755   468008.64598  4601077.26014
Monte Generoso                    4390113.33388   696884.17197  4561132.95496
```

⇒ eventually conversion to:

ellipsoidal coordinates and heights in ETRF93

```
$$EL ellipsoidal coordinates in ETRF93                           07.04.1999 15:59
Zimmerwald                          7 27 54.984923   46 52 37.541533      947.1511
Chrischona                          7 40  6.983077   47 34  1.385301      504.9275
Pfaender                            9 47  3.697719   47 30 55.172799     1089.3764
La Givrine                          6  6  7.326361   46 27 14.690021     1258.2466
Monte Generoso                      9  1 11.429926   45 55 49.983503     1741.2136
```